Griffiths 3.31.

$$\mathcal{R}_\pm^2 = r^2 + \left(\frac{d}{2}\right)^2 \mp r d\cos\theta$$

$$\frac{1}{\mathcal{R}_\pm} = \frac{1}{\sqrt{r^2 + \left(\frac{d}{2}\right)^2 \mp r d\cos\theta}} \qquad \text{expand in } \frac{d}{r}.$$

$$\cong \frac{1}{r}\left[1 \mp \frac{d}{r}\cos\theta\right]^{-1/2}$$

$$\Rightarrow \left[1 \mp \frac{d}{r}\cos\theta\right]^{-1/2}\Big|_0 = 1$$

$$= \left[-\frac{1}{2}\right]\left[1 \mp \frac{d}{r}\cos\theta\right]^{-3/2}(\mp\cos\theta).$$

$$|_0 = \left(-\frac{1}{2}\right)(\mp\cos\theta)$$

$$= \pm\frac{1}{2}\cos\theta$$

$$\frac{3}{4}\left[1 \mp \frac{d}{r}\cos\theta\right]^{-5/2}\left[\cos^2\theta\right]\Big|_0 = \frac{3}{4}\cos^2\theta$$

$$-\frac{5}{2}\frac{3}{4}\left[1 \mp \frac{d}{r}\cos\theta\right]^{-7/2}\left[\mp\cos^3\theta\right]\Big|_0 = -\frac{15}{8}\left[\mp\cos^3\theta\right]$$

$$\Rightarrow \frac{1}{\mathcal{R}_\pm} \cong 1 \pm \frac{1}{2}\cos\theta\frac{d}{r} + \frac{3}{8}\cos^2\theta\left(\frac{d}{r}\right)^2 \pm \frac{15}{8}\frac{5}{2}\cos^3\theta\left(\frac{d}{r}\right)^3.$$

$$\cong \boxed{1 \pm \frac{1}{2}\cos\theta\frac{d}{r} + \frac{3}{8}\cos^2\theta\left(\frac{d}{r}\right)^2 \pm \frac{5}{16}\cos^3\theta\left(\frac{d}{r}\right)^3 + \cdots}$$

quadrupole-related ↑ octopole-related ↑

$$\cos^2\theta\frac{d^2}{4} = z^2 \Rightarrow \text{quadrupole: } \frac{1}{2}\frac{z^2}{r^2}$$

Griffiths 3.31.

$$\frac{1}{\mathcal{R}_\pm} \cong \frac{1}{r}\left[1 \pm \cancel{\frac{1}{2}\cos\theta}\left(\frac{d}{2r}\right) + \frac{3}{2}\cos^2\theta\left(\frac{d}{2r}\right)^2 \pm \frac{5}{2}\cos^3\theta\left(\frac{d}{2r}\right)^3 + \cdots\right]$$

$\Rightarrow$ monopole

$$\Rightarrow \frac{1}{r} \pm \cos\theta\,\frac{d}{2}\,\frac{1}{r^2} + \frac{3}{2}\cos^2\theta\,\frac{d^2}{4}\,\frac{1}{r^3}.$$

$$\Rightarrow \frac{1}{r} \pm \boxed{\cos\theta\,\frac{d}{2}}\,\frac{1}{r^2} + \boxed{\frac{3}{8}\cos^2\theta\,d^2}\,\frac{1}{r^3} \pm \boxed{\frac{5}{16}\cos^3\theta\,d^3}\,\frac{1}{r^4}.$$

$\Downarrow$ $\qquad$ $\Downarrow$

$\frac{3}{2}\cos^2\theta\left(\frac{d}{2}\right)^2 \qquad \frac{5}{2}\cos^3\theta\left(\frac{d}{2}\right)^3$

For Φ $\frac{1}{\mathcal{R}_+} - \frac{1}{\mathcal{R}_-}$, we have

$$q\left[\frac{1}{\mathcal{R}_+} - \frac{1}{\mathcal{R}_-}\right] \cong \cancel{\cos\theta}\,\cos\theta\,qd\,\frac{1}{r^2} + \frac{5}{8}\cos^3\theta\,qd^3\,\frac{1}{r^4}$$