

Griffiths 3.31.

$$r_{\pm}^2 = r^2 + \left(\frac{d}{2}\right)^2 \mp r d \cos \theta$$

$$\frac{1}{r_{\pm}} = \frac{1}{\sqrt{r^2 + \left(\frac{d}{2}\right)^2 \mp r d \cos \theta}} \quad \text{expand in } \frac{d}{r}$$

$$\approx \frac{1}{r} \left[1 \mp \frac{d}{r} \cos \theta \right]^{-1/2}$$

$$\Rightarrow \left[1 \mp \frac{d}{r} \cos \theta \right]^{-1/2} \Big|_0 = 1$$

$$= \cancel{\left[1 \mp \frac{d}{r} \cos \theta \right]}^{-3/2} \left(-\frac{1}{2} \right) \left[1 \mp \frac{d}{r} \cos \theta \right] \left(\mp \cos \theta \right)$$

$$\Big|_0 = \left(-\frac{1}{2} \right) \left(\mp \cos \theta \right)$$

$$= \pm \frac{1}{2} \cos \theta$$

$$\frac{3}{4} \left[1 \mp \frac{d}{r} \cos \theta \right]^{-5/2} \left[\cos^2 \theta \right] \Big|_0 = \frac{3}{4} \cos^2 \theta$$

$$-\frac{5}{2} \frac{3}{4} \left[1 \mp \frac{d}{r} \cos \theta \right]^{-7/2} \left[\mp \cos^3 \theta \right] \Big|_0 = -\frac{15}{8} \left[\mp \cos^3 \theta \right]$$

$$\Rightarrow \frac{1}{r_{\pm}} \approx 1 \pm \frac{1}{2} \cos \theta \frac{d}{r} + \frac{3}{8} \cos^2 \theta \left(\frac{d}{r}\right)^2 \pm \frac{15}{16} \cos^3 \theta \left(\frac{d}{r}\right)^3$$

$$\approx \boxed{1 \pm \frac{1}{2} \cos \theta \frac{d}{r} + \frac{3}{8} \cos^2 \theta \left(\frac{d}{r}\right)^2 \pm \frac{5}{16} \cos^3 \theta \left(\frac{d}{r}\right)^3 + \dots}$$

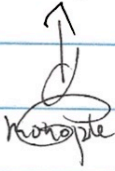
quadrupole-related

octopole-related.

$$\cos^2 \theta \frac{d^2}{4} = z^2 \Rightarrow \text{quadrupole: } \frac{1}{2} \frac{z^2}{r^2}$$

Gustafsson 3.31.

$$\frac{1}{\mu_{\pm}} \cong \frac{1}{r} \left[1 \pm \frac{1}{2} \cos \theta \left(\frac{d}{2r} \right) + \frac{3}{2} \cos^2 \theta \left(\frac{d}{2r} \right)^2 \pm \frac{5}{2} \cos^3 \theta \left(\frac{d}{2r} \right)^3 + \dots \right]$$

\Rightarrow 

 monopole

$$\Rightarrow \frac{1}{r} \pm \cos \theta \frac{d}{2} \frac{1}{r^2} + \frac{3}{2} \cos^2 \theta \frac{d^2}{4} \frac{1}{r^3}$$

$$\Rightarrow \frac{1}{r} \pm \boxed{\cos \theta \frac{d}{2}} \frac{1}{r^2} + \boxed{\frac{3}{8} \cos^2 \theta d^2} \frac{1}{r^3} \pm \boxed{\frac{5}{16} \cos^3 \theta d^3} \frac{1}{r^4}$$

$$\Downarrow \quad \Downarrow$$

$$\pm \quad \frac{3}{2} \cos^2 \theta \left(\frac{d}{2} \right)^2 \quad \frac{5}{2} \cos^3 \theta \left(\frac{d}{2} \right)^3$$

For $\ominus \frac{1}{\mu_+} - \frac{1}{\mu_-}$, we have

$$q \left[\frac{1}{\mu_+} - \frac{1}{\mu_-} \right] \cong \cancel{\cos \theta} \cos \theta q d \frac{1}{r^2} + \frac{5}{8} \cos^3 \theta q d^3 \frac{1}{r^4}$$